



Fermi National Accelerator Laboratory

FERMILAB-Pub-86/109-A
July 1986

LEPTONIC AND HADRONIC MASS SCALES - A COSMIC CONNECTION?

H. Fritzsch

Sektion Physik der Universität München

and

Max-Planck-Institut für Physik und Astrophysik

- Werner Heisenberg Institut für Physik -

München, Germany

and

David N. Schramm

University of Chicago

Chicago, IL 60637

and

Fermi National Accelerator Laboratory

Batavia, IL 60510



Today the world of elementary particles is described in terms of a number of different mass scales. The smallest mass scale (apart from zero) known experimentally is the electron mass $m_e = 0.511 \text{ MeV}$. There could be smaller mass scales given by the masses of the various neutrinos (in case, it turns out that neutrinos are not massless). In the standard electroweak theory the lepton and quark masses are essentially arbitrary parameters and related to the spontaneous breakdown of the electroweak gauge symmetry. The various lepton and quark masses are somewhat in between the electron mass and the t-quark mass (possibly of the order of 30 - 40 GeV).

Besides the lepton and quark masses the weak interaction mass scale given by the masses of the weak bosons (of order 100 GeV) enters, as well as the hadronic mass scale determined by the QCD cutoff parameter Λ_c (of order 150 MeV). It is interesting to note that the QCD scale parameter Λ_c (which determines e.g. the nucleon mass) and the various lepton and quark masses are, roughly speaking, of comparable order of magnitude, while conceptually both are of a completely different origin and could in principle differ by many orders of magnitude. In this note we should like to point out that this may not be an accident, but a consequence of an intrinsic connection between the QCD mass scale and the lepton-quark mass scales, which reflects itself in specific properties of the cosmological evolution around a redshift of 1000.

In the standard hot Big Bang model of the cosmological evolution the onset of the formation of large structures (clusters, galaxies) is marked by two different events:

- a) The decoupling of matter and radiation due to the binding of electrons and protons to nuclei in atoms.
- b) The transition from a radiation dominated universe to a universe whose energy density is dominated by the nonrelativistic matter. In fact the equality of radiation density and baryonic mass density occurs at the same time as decoupling to within the accuracy these quantities are known.

There is no direct connection between those two cosmic events. The decoupling of radiation and matter occurs once the radiation temperature drops sizably below about one tenths of the Rydberg energy $m_e \alpha^2 / 2 \approx 13.6 \text{ eV}$:

$$T_{\text{dec}} \approx (m_e \cdot \alpha^2 / 2) 10^{-1} \quad (1)$$

The transition from the radiation dominated universe to a matter nuclear dominated one is determined by the nucleon mass and by the ratio n_B/n_γ (n_B : density of baryons). Yet it turns out that both events occur at about the same time

corresponding to a redshift of order 1000, and to a temperature of the order of 10^4 K. On the other hand the coincidence of both events is of great importance for the structure of the universe today. In particular it is important for the formation of extended structures rather early in the cosmic evolution. (If $\Omega = \rho/\rho_{\text{crit}} \approx 1$ (ρ : matter density), the dominant part of the matter is provided by nonbaryonic material. One expects $\Omega_{\text{baryons}} \sim 0.1$, i.e. the transition is actually a transition from a radiation dominated to a baryon dominated universe with the actual transition from radiation to matter domination occurring a factor ~ 10 earlier in redshift.

The transition from radiation dominance to baryon dominance takes place if the radiation energy density ρ_Y and the baryon energy density are equal:

$$\rho_Y \approx \rho_b. \quad (2)$$

$$\rho_Y = \frac{\pi^2}{15} T_c^4 \approx \rho_b = \rho_b^0 \cdot \left(\frac{T_c}{T_0}\right)^3$$

(T_0 : present temperature (~ 2.7 K), T_c : transition temperature), ρ_b^0 : present baryonic energy density)

One finds from eq (1.2):

$$T_c \approx \frac{15}{\pi^2} \cdot \frac{\rho_b^0}{T_0^3} \quad (3)$$

The present baryonic energy density ρ_b^0 is given by:

$$\rho_b^0 = M \cdot n_b^0 = M \cdot n_Y^0 \cdot \left(\frac{n_b}{n_Y}\right) = M \cdot n_Y^0 \cdot \eta \quad (4)$$

(n_b : baryon density, n_Y : photon density, M : nucleon mass, $\eta = n_b/n_Y \approx 5 \cdot 10^{-10}$).

Thus we obtain for the critical temperature:

$$T_c \approx \frac{15}{\pi^2} \cdot M \cdot \eta \cdot \frac{n_Y^0}{T_0^3} = \frac{15}{\pi^2} \cdot \frac{2.4}{\pi^2} \eta M = 0.37 \eta M \quad (5)$$

where we have used the relation:

$$n_Y = \frac{2.4}{\pi^2} T^3 \quad (6)$$

We require $T_c \approx T_{dec}$ and obtain:

$$\frac{\alpha^2 m_e}{20} \approx 0.37 \eta M \quad (7)$$

or:

$$\frac{\alpha^2 m_e}{M} \approx 7.4 \eta \quad (8)$$

In grand unified theories the ratio η is calculable in terms of the decay parameters, of heavy bosons in particular by the parameters describing CP violation in the baryon non-conserving interaction. In most realistic models it does not depend on α .

In ref. (1) it was pointed out that the coincidence $T_{dec} \approx T_c$ implies a relationship between the parameter of CP violation and the finestructure constant α . However as noted above such a relationship is not currently part of realistic grand unified models. Another way to eliminate the "coincidence" would be if there is a relationship between both the electron mass and the proton mass and the fine structure constant so as to obtain eq.(8). In this paper we should like to focus on such implications. As one can see from eq. (8), the smallness of the ratio m_e/M is related to the smallness of η . Thus far the origin of the electron mass is unknown. However in grand unified theories the nucleon mass, i.e. the QCD scale parameter Λ , is related to the mass M_X , the mass of the hypothetical X boson, setting the scale at which the grand symmetry is restored:

$$M = e^{-\frac{a}{\alpha}} M_X. \quad (9)$$

In the minimal SU(5) model (see e.g. ref. (2)) one has $a = \frac{\pi}{11}$.

Thus we can rewrite eq. (9) as follows:

$$\frac{\alpha^2 m_e}{e^{-\frac{a}{\alpha}} M_X} = 7.4 \eta \quad (10)$$

The l.h.s. of eq. (10) depends on α in a sensitive way. Even a slight change of α will change the nucleon mass drastically, and relation (10) will not be true anymore, hence the coincidence $T_{dec} \approx T_c$ is not present, unless the mass M_X as well as the electron mass change likewise. (For example, if α is changed from $1/137$ to $1/100$, the l.h.s. of eq. (10) is increased by a factor 10^6 , if m_e and M_X remain unchanged.)

Of course, it could be that the coincidence $T_{\text{dec}} \approx T_c$ is merely a reflection of the fact that the finestructure constant α has a very specific value, and the coincidence does not occur for any other value of α . However we find such a point of view unsatisfactory; it implies a finetuning of α . It would be more natural to suppose that relation (10) is fulfilled for a wide range of values of α . This is only possible if the mass parameters m_e and M_X depend on α , and it is this (hypothetical) dependence we should like to explore.

In a complete theory of all particle physics phenomena only one mass scale is expected to occur, the Planck mass $M_p \approx 10^{19}$ GeV. All other masses, e.g. the lepton and quark masses as well as M_X , are expected to be functions of M_p . Relation (10) implies in particular:

$$\frac{\alpha^2 m_e}{e^{-\frac{a}{\alpha}} M_X} = \text{const.} \quad (11)$$

Since the electron mass is extremely small in comparison to M_p , one expects that the relation between M_p and m_e involves an exponential, and we shall assume:

$$m_e = \text{const.} \cdot e^{-\frac{a}{\alpha}} M_p \quad (12)$$

On the other hand M_X and M_p do not differ by many orders of magnitude. A possible ansatz would be:

$$M_X = \text{const.} \cdot \alpha^2 \cdot M_p, \quad (13)$$

placing M_X to be of the order of 10^{15} GeV, in accordance with the expectations in grand unified theories.

Inserting both eq. (12) and eq. (13) in relation (11), one finds that the l.h.s. of (11) is indeed independent of α . If the functional dependences eq. (12) and eq. (13) are correct, the coincidence $T_{\text{dec}} \approx T_c$ occurs for a wide range of α -values - no finetuning problem exists.

Relations similar to (12) might hold for all lepton and quark masses:

$$m_{l,q} \sim \text{const.} \cdot e^{-a/\alpha} M_p. \quad (14)$$

Since both the QCD mass scale Λ_c and the lepton and quark masses do depend on the same (small) exponential factor, it becomes plausible why the hadronic mass scale and the various lepton and quark masses are roughly of the same order of magnitude. The cosmological arguments discussed here suggest a common origin of all mass scales in physics.

Acknowledgement:

This work was supported in part by the U.S. Department of Energy at the University of Chicago and the Deutsche Forschungsgemeinschaft (DFG- contract Fr 412/6-2).

References

1. E. Vishniac and D.N. Schramm 1986
Comments on Nuclear and Particle Physics (In press).
2. P. Langacker, Phys. Rep. 72 C 1981, 185.